AN APPLICATION OF THE LPI SOLUTION TECHNIQUE IN THE NWS FLDWAV MODEL

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Abstract

A Local Partial Inertia (LPI) technique has been developed as part of the National Weather Service (NWS) dynamic flood routing model (FLDWAV) to enhance its capability to model unsteady flows in the subcritical/supercritical mixed-flow regimes especially in the near critical range of the Froude number. By applying a simple numerical filter (σ) to the inertial terms in the unsteady flow momentum equation according to the local Froude number, the LPI technique retains the essential accuracy associated with dynamic routing models and provides stable numerical solutions for mixed flows for the four-point implicit numerical scheme used in the FLDWAV model. This paper briefly introduces the LPI technique and presents an application of the technique to a dam-break-induced flood wave which is routed through a river reach which experiences mixed-flow.

Introduction

The NWS FLDWAV model is a generalized flood routing model which is based on an implicit, weighted, four-point, nonlinear, finite-difference solution of the one-dimensional unsteady flow (Saint-Venant) equations. FLDWAV combines the capabilities of the popular NWS DAMBRK and DWOPER models (Fread, 1993) and provides additional features such as: (1) a multiple levee overtopping/crevasse option, (2) a multiple-reach routing algorithm which enables the application of different routing techniques (implicit, explicit, level-pool, diffusion, etc.) to specified subreaches, and (3) a new network solution algorithm for any dendritic river system. A new feature of FLDWAV, presented herein, uses an LPI solution technique for modeling "mixed" (subcritical and/or supercritical) flows in specified subreaches.

Among the various capabilities of the FLDWAV model, dam-breaching analysis and dynamic routing of the dam-break induced flood wave are two very useful tools in dam safety related studies and applications. One challenge in modeling the dam-break-induced unsteady flows is that mixed flow regimes and moving interfaces

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(subcritical/supercritical or conversely) may occur in certain situations, e.g., the flow from a near-instantaneous breach of a large dam is routed through a channel with a mild slope. When modeling unsteady flows, the dynamic technique using the four-point implicit numerical scheme tends to be less numerically stable than the diffusion (zero inertia) routing technique for certain mixed flows, especially in the near critical range of the Froude number (F_r) or mixed flows with moving supercritical/subcritical interfaces. It has been observed that the diffusion technique, which eliminates the two inertial terms in the momentum equation, produces stable numerical solutions for flows where F_r is in the range of critical flow ($F_r=1.0$). To take advantage of the stability of the diffusion method, and retain the accuracy of the dynamic method, a "local partial inertia" (LPI) technique has been developed within the NWS FLDWAV model in which a numerical filter (σ) modifies the extent of contribution of the inertial terms in the momentum equation so that its properties vary from dynamic to diffusion.

LPI Technique

The FLDWAV model is based on a four-point, implicit, numerical, solution scheme of the Saint-Venant unsteady flow equations (Fread, 1993):

$$\frac{\partial Q}{\partial x}+\frac{\partial (A+A_0)}{\partial t}-q=0 \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial (\beta Q^2/A)}{\partial x}+gA\left(\frac{\partial h}{\partial x}+S_f+S_e\right)+L=0 \tag{2}$$

in which t is time, x is distance along the longitudinal axis of the waterway, h is the water surface elevation, A is the active cross-sectional area of flow, A_0 is the inactive (off-channel storage) cross-sectional area of flow, q is the lateral inflow or outflow, β is the coefficient for nonuniform velocity distribution within the cross section, g is the gravity constant, S_f is the friction slope, S_e is the slope due to local expansion-contraction (large eddy loss), and L is the momentum effect of lateral flow ($L=-qv_x$ for lateral inflow, where v_x is the lateral inflow velocity in the x-direction; $L=-qQ/(2A)$ for seepage lateral outflows; $L=-qQ/A$ for bulk lateral outflows). The first two terms in the momentum equation, Eq.(2), are the inertial terms.

In the LPI technique (Fread, Jin, and Lewis, 1996), the momentum equation, Eq. (2), is modified by a numerical filter, σ, so that the inertial terms are partially or totally omitted according to the local flow condition as defined by the local Froude number. The modified momentum equation and the numerical filter are:

$$\sigma\left[\frac{\partial Q}{\partial t}+\frac{\partial (\beta Q^2/A)}{\partial x}\right]+gA\left(\frac{\partial h}{\partial x}+S_f+S_e\right)+L=0 \tag{3}$$

$$\sigma = \begin{cases} 1.0 - F_r^{\,m} & (F_r \le 1.0; \quad m \ge 1) \\ 0 & (F_r > 1.0) \end{cases} \tag{4}$$

in which F_r is the Froude number and m is a user specified constant. Figure 1 shows the variation of the filter, σ, with F_r and with the value of m. The σ filter, which depends on F_r, has a variation that ranges from a linear function to the Dirac delta function. Since the Froude number is determined at each computational point (each cross section and instance of time), σ is a "local" parameter. Therefore, portions of the routing reach with low Froude numbers will be modeled with all or essentially all of the inertial terms included, while those portions with F_r values in the vicinity of critical flow will be modeled with "partial inertial" effects included; supercritical flows ($F_r > 1$) will be modeled with no inertial effects. It is found that smaller values of m tend to stabilize the solution in some cases while larger values of m provide more accuracy. By using the σ filter, the FLDWAV model automatically changes from a dynamic model to a diffusion model and takes advantage of the stability of the diffusion model for those flows with F_r near the critical value of 1.0.

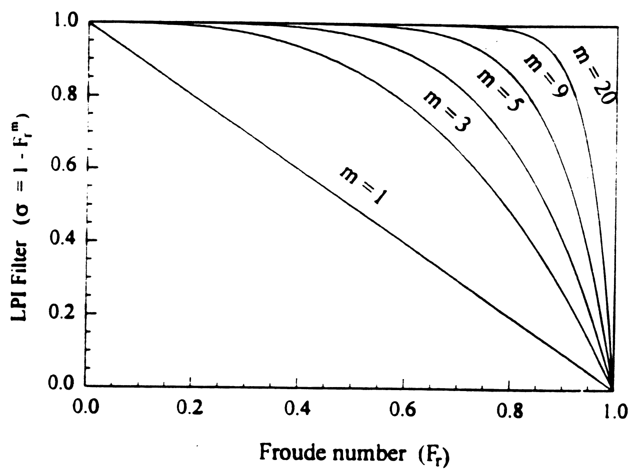

Figure 1 The LPI Filter

The error properties of the LPI technique, which totally or partially omit the inertial terms of the momentum equation, have been theoretically analyzed and numerically tested. It has been shown that the proportional contribution of the inertial terms, noted as IT (which is the inertial terms divided by the water surface slope), to the total momentum equation depends on the flow Froude number and another dimensionless parameter, ϕ. The term, IT, can be shown to be related to the Froude number (F_r) and ϕ as follows:

$$IT = \frac{-0.5\ F_r^2}{1 + 1.5\ \phi\ F_r^3} \tag{5}$$

where:

$$\phi = \frac{n^2 g^{3/2} y^{1/6}}{\lambda^2 \partial y / \partial t} \tag{6}$$

in which n is the Manning's resistance coefficient, and λ is the constant in Manning's equation ($\lambda = 1.49$ for English system of units and $\lambda = 1.0$ for SI units). The new parameter (ϕ), reflects the flow's unsteadiness and hydraulic condition. Further analysis (Fread, Jin, and Lewis, 1996) has shown that IT is a very small term (usually less than 4% of the total momentum equation) and that IT decreases rapidly as the ϕ value increases and F_r approaches 1.0; therefore, Eq. (2) is very closely approximated by Eq. (3) in most unsteady flow conditions.

Figures 2 and 3 show some test results of the computational errors for the LPI technique. The errors are considered as differences between the results of using the complete momentum equation (dynamic routing) and the results of using the LPI modified equation. Numerical experiments compare the results from both methods for a broad range of unsteady flow conditions, and two kinds of errors are examined. The error E_{pk} (%), as shown in Figure 2, is the maximum normalized error in the computed peak profiles; the error E_{rms} (%), as shown in Figure 3, is the normalized root-mean-square (RMS) error in the computed hydrographs. These results show that the overall errors in using the LPI technique are very small (less than 2%) for most situations ($\phi > 10$) and less than 6% for special situations ($5 \leq \phi \leq 10$) which are only applicable for near instantaneous large dam-failure induced floods in channels of very flat bed slopes.

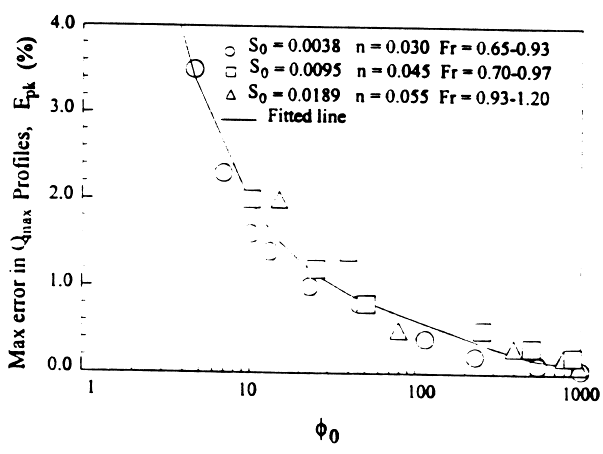

Figure 2 Errors in the computed peak flow

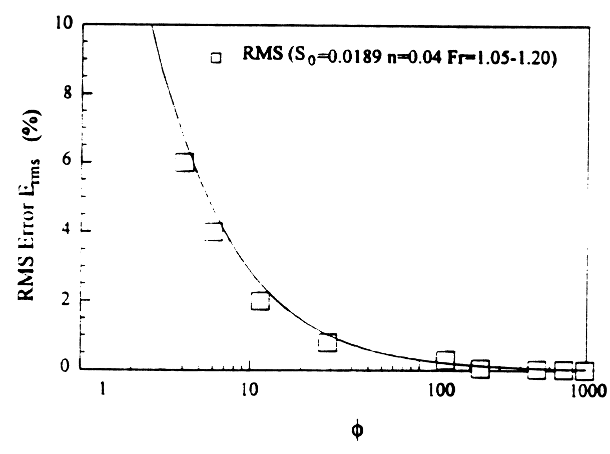

Figure 3 RMS errors in computed hydrographs

Application

The LPI-enhanced FLDWAV model is applied to a dam-break situation. A 120 ft high dam is breached within 0.5 hours. The reservoir stores about 1.15×10^6 acre-ft of water; and level-pool storage routing and the dynamic routing are used upstream and downstream of the dam, respectively. The dam-break-induced flood wave travels through a 37.2-mile reach of an extremely non-prismatic channel, with a bottom slope varying from 0.0230 (124 ft/mile) immediately below the dam to 0.0054 (28 ft/mile) at the downstream end of the reach, and the Manning's n varies from 0.07 upstream to 0.04 downstream. Mixed and near-critical flow occurs in some steeper portions of the reach when the flow changes from the initial low flow to its peak, and subcritical/supercritical moving interfaces are present as the flood peak moves downstream.

This near-critical mixed flow and moving interface causes numerical stability problems when using the conventional four-point implicit scheme. A special mixed-flow algorithm (Fread, 1992) developed earlier in the FLDWAV model which divides the entire routing reach into a series of subcritical and supercritical subreaches by searching for the transitional control points at each time step also failed for this nearly critical mixed-flow situation. Figure 4 shows computed water surfaces for the reach between $x=25$ and $x=30$ miles at the times $t=1.20$ hour and $t=1.30$ hour at which the special mixed-flow algorithm failed. The results from three techniques are shown in this figure, they are: (1) the special mixed-flow algorithm, (2) the new LPI technique presented herein and (3) a characteristic-based, upwind, explicit, dynamic routing technique (Jin and Fread, 1995) which is available in the FLDWAV model to simulate nearly instantaneous dam-failure-induced flood waves and near-critical mixed flows. The explicit scheme has been tested successfully for its performance in modeling near-instantaneous dam-break waves and mixed flows. It is observed that the LPI and the explicit techniques simulate the wave front similarly while the special mixed-flow method generates very different surface profiles and becomes unstable before it fails.

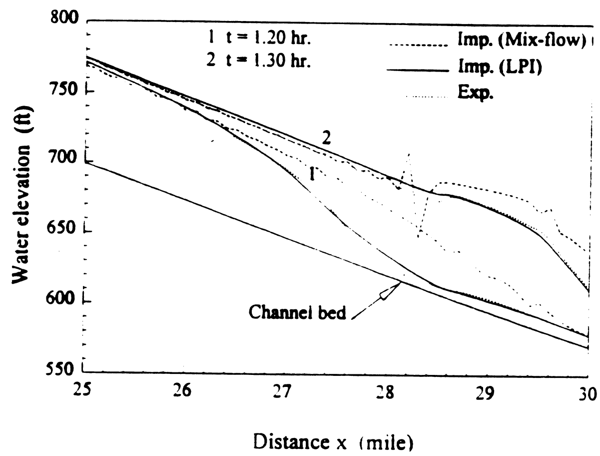

Figure 4 Computed water surfaces

Using the new LPI technique with $m=5$, the FLDWAV model produces stable and smooth solutions for the dam-break induced flood wave simulation. Some computational results are shown in Figures 5 through 8. In these figures, the results from the LPI technique are also compared with those obtained from the explicit scheme.

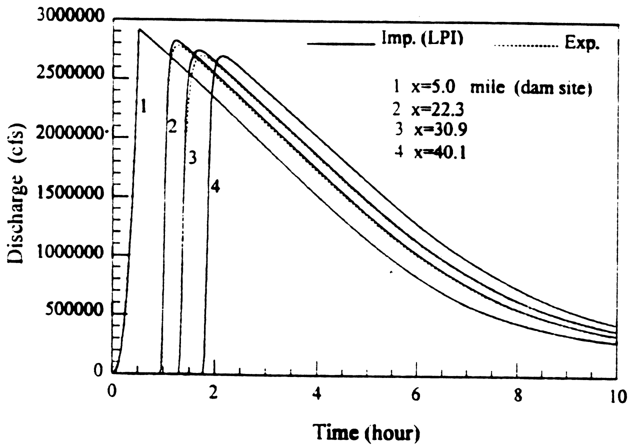

Figure 5 Computed hydographs

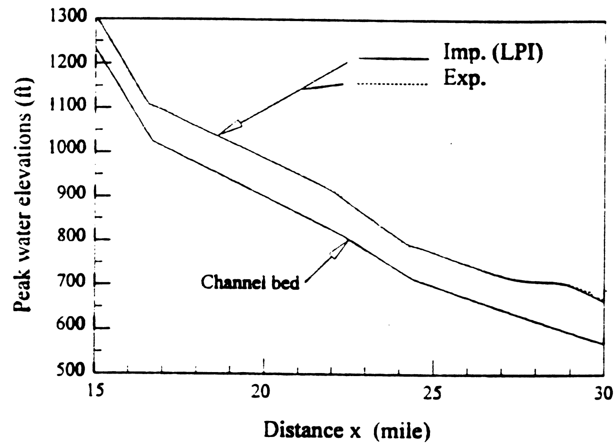

Figure 6 Computed water stage peak profils

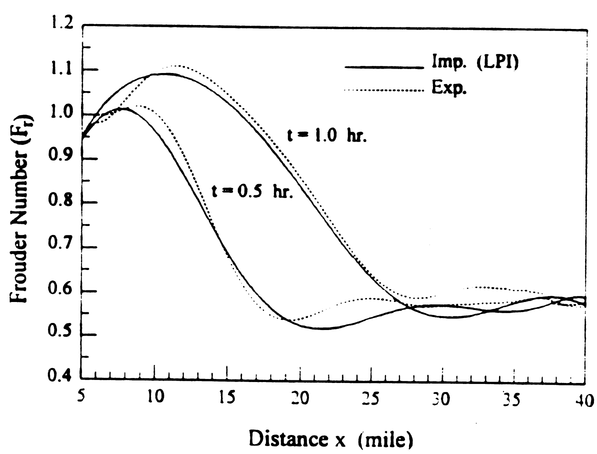

Figure 7 Froude number distributions

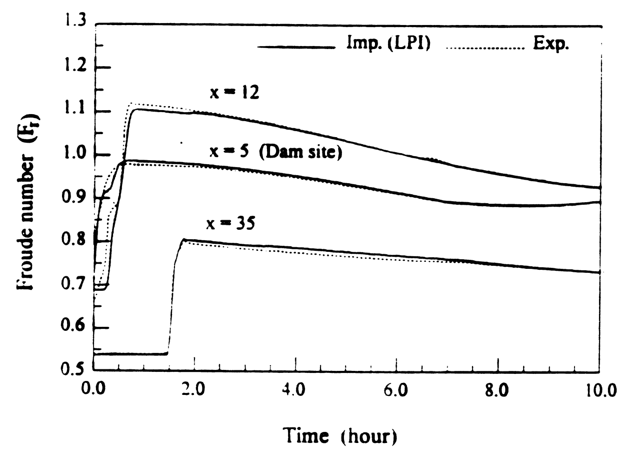

Figure 8 Froude number variations

Figure 5 compares the LPI-computed dam-breach outflow hydrograph (at dam site x = 5 miles) and hydrographs at three locations along the routing reach with those results from the explicit technique. The close agreement between the more computationally efficient LPI technique and the explicit technique suggest that these results are reliable.

Figure 6 shows the computational results of the peak water stage profile along part of the routing reach. Close agreement between the LPI and the explicit results are also observed.

One of the major difficulties in modeling this kind of mixed flow is that there is often a subcritical/supercritical moving interface associated with the advancing flood peak. The "mixed" flow regime changes not only with the time but also with space, and the regime is often in the range of near-critical flow (Froude number approaching 1.0) which, by its nature, is physically unstable. Figure 7 shows the instantaneously computed Froude number distributions along the routing reach for two specified times (t=0.5 and t=1.0 hours). The interface between subcritical and supercritical flow is indicated by the point in the distribution where the Froude number is equal to one. It can be seen that the LPI technique is capable of modeling the interface which moves from mile x=8 at t=0.5 hr to its location at mile x=16 at t=1.0 hr.

For a given location, the flow regime may change as the dam-break-induced flood wave passes. Figure 8 show some computed results of the Froude number variation with time for three locations (x=5, 12, and 35 miles). The flow regime at the dam site is always subcritical, but near critical ($F_r=1$). The flow regime at mile x=12 is seen to change from subcritical to supercritical and then back to subcritical after the flow peak passes. The flow regime at mile x=35 is always subcritical.

These results indicate that the LPI technique works well in modeling unsteady flow of subcritical/supercritical mixed-flow regimes with moving interfaces and the near critical flow regime.

Conclusion

The LPI technique, which filters the inertial terms in the one-dimensional unsteady flow momentum equation according to the local Froude number, increases the stability of the FLDWAV model to simulate near-critical subcritical/supercritical mixed flows, including supercritical/subcritical moving interfaces, while retaining the accuracy of dynamic modeling for subcritical flows. The application example presented in this paper, and the comparison between the results of the LPI technique and the characteristic-based upwind explicit technique, show that the LPI technique does very well in modeling mixed flows.

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